



**EXPONENTIAL DEPENDENT DEMAND RATE ECONOMIC PRODUCTION
QUANTITY (EPQ) MODEL WITH FOR REDUCTION DELIVERY POLICY**

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Abstract

This paper derives, production inventory model for exponential dependent demand and cost reduction delivery policy. Production rate is proportional to demand rate. Mathematical model is presented to find optimal order quantity and total cost. Numerical example is provided to validate the optimal solution. Sensitivity analysis is carried out to distinguish the change in the optimal solution with respect to the variation of one parameter at a time. Mathematica software is used to find the numerical results.

Keywords: *Production, inventory; optimality; exponential demand rate*



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Introduction

The traditional EPQ (Economic Production Quantity) model considers that the production products are all in full strength. However, in real life it is not valid in real life due to imperfect quality items. The EPQ (Economic Production Quantity) are used for finding the optimal order quantity. In this paper demand rate and production rate both considered to be exponential time-dependent. The objective of this research is to investigate the effect of total cost with the variation of key parameters. Harries [1] presented inventory model and derived some useful results. After that Taft [2] presented EPQ (Economic Production Quantity) formula. Rosenblatt and Lee [3] studied the effects of an imperfect production process on the optimal production

cycle time for the traditional economic manufacturing quantity (EMQ) model. Several inventory models in the inventory literature were presented considering variable demand. Silver and Meal [4] were first developed an economic order quantity model for the case of the time dependent demand. Teng *et al.* [5] presented an EOQ model for increasing demand in a supply chain with trade credits. Teng *et al.* [6] established inventory model with increasing demand under trade credit financing. Khanra *et al.* [7] presented an inventory model for deteriorating item with time-dependent demand. Large number of research papers presented by authors like Jalan *et al.* [8], Mitra *et al.* [9] , Dave and Patel [10] , Jalan and Chauduri [11] , in this direction. Hou [12] presented an economic production model with imperfect production process, in which the setup cost is a function of capital expenditure. Cardenas- Barron [13] developed an economic production quantity inventory model with planned backorders for deteriorating the economic production quantity for a single product. Samanta and Roy [14] established a continuous production inventory model of deteriorating item with shortages and considered that the production rate is changed to another at a time when the inventory level reaches a prefixed level. Cardenas- Barron [15] dealt an economic production quantity model with production capacity limitation and breakdown with immediate rework. Recently, Liu *et al.* [16] delt the problem of a production system that can produce multiple products but also subject to preventive maintence at the setup times of some products. Krishnamoorthi *et al.* [17] presented a single stage production process where defective item produced are reworked and two models of rework process are considered , an EOQ model for with and without shortages. Ghiami and Williams [18] established a production- inventory model in which a manufacturer is delivering a deteriorating product to retailers. Taleizadeh *et al.* [19] developed a vendor managed inventory for two level supply chain comprised of one vendor and several non- competing retailers, in which both the raw material and the finished product have different deterioration rates. Donaldson [20] established an EOQ model for trend demand. Organization of the manuscript is as follows: Assumptions and notion is provided in section 2. We provide mathematical formulation in the next section 3. In section 4 optimal solutions are discussed. Numerical example is given in section 5. Sensitivity analysis with respect to parameters is discussed in section 6. Conclusion and future research directions are discussed in the last section 7.

Assumptions and Notation

The following assumptions are made throughout the manuscript:

(i).The production rate is proportional to demand rate, i.e. $P \propto D$, $P = aD$,

Where a is proportionality constant and $a > 1$

(ii).Demand rate is time dependent i.e. $D \equiv \lambda_0 e^{\alpha t}$ where λ_0 is the initial inventory and $0 < \alpha < 1$

(iii).Items are produced and added to the inventory.

(iv)Two rates of production are considered.

(v).The production rate is always greater than demand rate.

In addition the following notations are used to form the inventory models:

$I(t)$	inventory level at any time 't'
P	production rate in units per unit time
D	demand rate in units per unit time
Q_l	on hand inventory level during $[0, T_l]$
Q	production quantity
Q^*	optimal production quantity
p	production cost per unit
h	holding cost per unit time
c	setup cost per setup
HC	holding cost per supplier
TC	total cost
TC^*	optimal total cost
T	cycle time
T_l	production time
PC	production cost
SC	setup cost

3. Mathematical Formulation

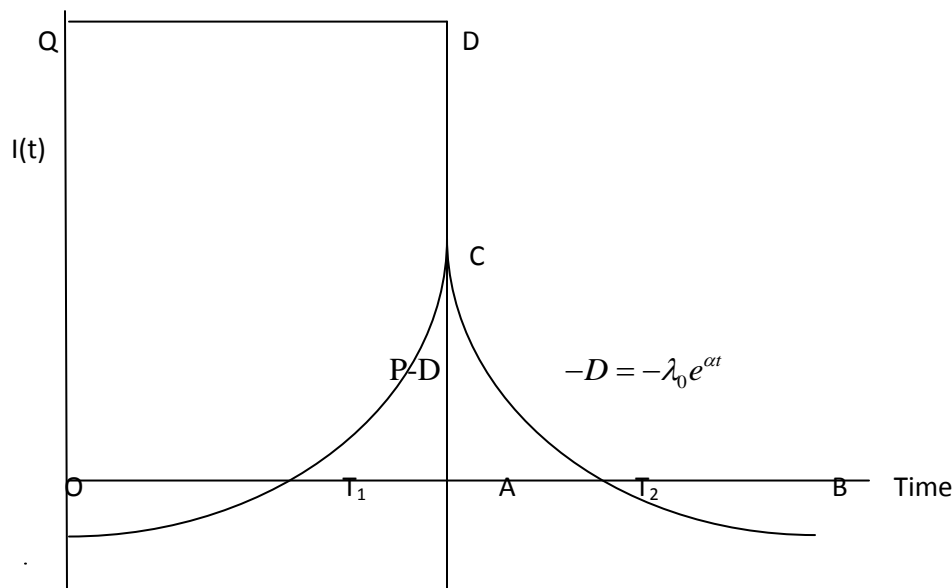


Fig. 1 Inventory vs time

During $t = 0$ to $t = T_1$, the inventory accumulates at a rate $P - D$. The production begins at O and ends at C . The consumption starts from $t = T_1$ and finished at $t = T_2$. The rate of change of inventory between $[T_1, T_2]$ is given by

$$\frac{dI(t)}{dt} = (a-1)\lambda_0 e^{\alpha t} \quad , \quad 0 < \alpha < 1, \quad 0 \leq t \leq T_1 \quad (1)$$

and
$$\frac{dI(t)}{dt} = -\lambda_0 e^{\alpha t} \quad , \quad 0 < \alpha < 1, \quad T_1 \leq t \leq T_2 \quad (2)$$

Solution of (1) and (2) with the condition $I(0) = 0$, $I(T_1) = Q$, $I(T) = 0$, and $T = T_1 + T_2$ is given by

$$I(t) = \frac{\lambda_0(a-1)}{\alpha} (e^{\alpha t} - 1) \quad , \quad 0 \leq t \leq T_1 \quad (3)$$

And
$$I(t) = \frac{\lambda_0}{\alpha} (e^{\alpha T} - e^{\alpha t}) \quad , \quad T_1 \leq t \leq T_2 \quad (4)$$

At $t = T_1$, Equations (3) and (4) are same i.e.

$$T_2 = (a-1) T_1 \quad (\text{approximately}) \quad (5)$$

At $t = 0$, the order quantity
$$Q = \frac{\lambda_0}{\alpha} (e^{\alpha T} - 1) \quad (6)$$

or $\alpha\lambda_0 T^2 + 2\lambda_0 T - 2Q = 0$, (approximately)

or
$$T = \frac{\sqrt{\lambda_0 + 2\alpha Q} - \sqrt{\lambda_0}}{\alpha\sqrt{\lambda_0}} \tag{7}$$

The total cost is calculated by considering setup cost, production cost and inventory holding cost:

1. Setup cost $SC = \frac{c}{T} = \frac{c\alpha\sqrt{\lambda_0}}{\sqrt{\lambda_0 + 2\alpha Q} - \sqrt{\lambda_0}}$ (8)

2. Production cost $PC = \frac{p\lambda_0}{2} \left\{ 1 + \frac{\sqrt{\lambda_0 + 2\alpha Q}}{\sqrt{\lambda_0}} \right\}$ (9)

3. Inventory holding cost $HC = \frac{h\lambda_0}{\alpha T} \left[(a-1) \int_0^{T_1} (e^{\alpha t} - 1) dt + \int_{T_1}^{T_2} (e^{\alpha T} - e^{\alpha t}) dt \right]$

$$= \frac{h\lambda_0}{\alpha T} \left[(a-1) \left(\frac{e^{\alpha T_1} - 1}{\alpha} - T_1 \right) + \left\{ e^{\alpha T} (T_2 - T_1) - \frac{e^{\alpha T_2} - e^{\alpha T_1}}{\alpha} \right\} \right]$$
 (10)

Total cost is the sum of *setup* cost, production cost and inventory cost i.e.

$$TC = \frac{c\alpha\sqrt{\lambda_0}}{\sqrt{\lambda_0 + 2\alpha Q} - \sqrt{\lambda_0}} + \frac{p\lambda_0}{2} \left\{ 1 + \frac{\sqrt{\lambda_0 + 2\alpha Q}}{\sqrt{\lambda_0}} \right\} + \frac{h\lambda_0}{2} \left\{ \frac{\alpha(aT_1^2 - T_2^2)\sqrt{\lambda_0}}{\sqrt{\lambda_0 + 2\alpha Q} - \sqrt{\lambda_0}} + (T_2 - T_1) \left(1 + \frac{\sqrt{\lambda_0 + 2\alpha Q}}{\sqrt{\lambda_0}} \right) \right\}$$
 (11)

4. Optimal Solution

The optimal solution is obtained by differentiating (11) with respect to Q , we get

$$\frac{dTC}{dQ} = \frac{\alpha\sqrt{\lambda_0}}{(\lambda_0 + 2\alpha Q)^{1/2}} \left[-\frac{\alpha \left\{ c + \frac{1}{2} h\lambda_0 (aT_1^2 - T_2^2) \right\}}{(\sqrt{\lambda_0 + 2\alpha Q} - \sqrt{\lambda_0})^2} + p + h(T_2 - T_1) \right]$$
 (12)

And $\frac{d^2TC}{dQ^2} > 0$ (13)

The optimal (minimum) production quantity $Q = Q^*$ is obtained by solving $\frac{dTC}{dQ} = 0$, we get

$$\alpha \left\{ c + \frac{1}{2} h\lambda_0 (aT_1^2 - T_2^2) \right\} - \{ p + h(T_2 - T_1) \} (\sqrt{\lambda_0 + 2\alpha Q} - \sqrt{\lambda_0})^2 = 0$$
 (14)

4. Numerical Example:

Let us consider the parameter values of the inventory system $a = 1.1$, $\lambda_0 = 500$, $\alpha = 0.2$, $c = 20$, $h = 50$, $p = 1000$ units, $T_1 = 0.4$ and $T_2 = 0.2$, in appropriate units. Substituting these values in (14) and (11), we get $Q = Q^* = 1069.54$ units and $TC = TC^* = \$ 591503.0$

6. Sensitivity Analysis

Sensitivity analysis is performed with the variation of different key parameters. Taking all the numerical values as mentioned in the above numerical example.

Table 1: Variation of production cost ‘p’, setup cost ‘c’ and holding cost ‘h’ on the optimal solution

<i>p</i>	<i>Q</i>	<i>TC</i>	<i>c</i>	<i>Q</i>	<i>TC</i>	<i>h</i>	<i>Q</i>	<i>TC</i>
1050	1115.91	624553.0	25	1068.60	591438.0	55	982.088	585196.0
1100	1161.11	657816.0	30	1067.66	591372.0	60	905.124	579578.0
1150	1205.24	691287.0	35	1066.72	591307.0	65	836.396	574513.0
1200	1248.41	724960.0	40	1065.77	591240.0	70	774.242	569899.0
1250	1290.69	758831.0	50	1063.89	591109.0	80	664.868	561737.0

All the above observations can be sum up as follows:

- Increase of production cost p leads increase in order quantity and increase in total cost. That is , change of p will lead positive change in Q and TC
- Increase in setup cost c leads decrease in order quantity and total cost. That is, change in c leads negative change in Q and TC
- Increase in holding cost h leads increase in order quantity a decrease in total cost. That is, change in h leads positive change in Q and negative change in TC

7. Conclusion

In this paper, we developed production inventory model with exponential time- dependent demand rate. Mathematical formulation is presented to find optimal order quantity and total cost.

A numerical example is provided to demonstrate the applicability of proposed model. From the sensitivity analysis we have obtained the following managerial phenomena.

- Increase of production cost leads increase in order quantity and increase in total cost
- Increase in setup cost leads decrease in order quantity and total cost
- Increase in holding cost leads increase in order quantity a decrease in total cost

This model discussed can be extended for several ways. For instance, we may extend the model for stock- dependent demand as well as ramp type demand. We could generalize the model for deteriorating items.

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